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ABSTRACT

A procedure is described that enables students to perform operations on fractions with a calculator, expressing the answer as a fraction. Patterns using paper-and-pencil procedures for each operation with fractions are presented. A microcomputer software program illustrates how the answer can be found using integer values of the numerators and denominators of fractions. An adaptation of this computer program is used to devise a method for using calculators in work with addition of fractions. First, some old algorithms and an explanation of Euclid's Algorithm are given as background for understanding the calculator procedure. Then the steps in that procedure are listed, stressing that multiplication, division, and subtraction of fractions can be performed, as well as addition. (MNS)

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A PROCEDURE USING CALCULATORS TO EXPRESS ANSWERS IN FRACTIONAL FORM

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Most calculator activities for operations on rational numbers described in texts, books, and articles result in the student performing the operations with numerals expressed in decimal form. Usually, the answer obtained in the activity is expressed in decimal form. Unfortunately, this is true even when the operation is performed on rational numbers expressed in fractional form. The primary purpose of this narrative is to describe a procedure that will enable students to perform number operations on rationals in fractional form with a calculator and express the answer as a fraction.

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In developing computational skills with fractions, we have employed patterns similar to the following:

Addition. If the fractions have a common denominator, write the sum of the numerators over the common denominator. For example:

$$\frac{2}{5} + \frac{1}{5} = \frac{2 + 1}{5} = \frac{3}{5}$$

If the fractions do not have a common denominator, find the LCD; rename the fractions and proceed as above. For example:

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{4 + 3}{6} = \frac{7}{6}$$

Subtraction. If the fractions have a common denominator, subtract the numerator of the second fraction from the first and write the difference over the common denominator. For example:

$$\frac{4}{7} - \frac{2}{7} = \frac{4 - 2}{7} = \frac{2}{7}$$

If the fractions do not have a common denominator, find the LCD; rename the fractions and proceed as above. For example:

$$\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{4 - 3}{6} = \frac{1}{6}$$

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Multiplication. Write the product of the numerators over the product of the denominators. For example:

$$\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$$

Division. Multiply the first fraction by the reciprocal of the second fraction, or as more commonly expressed, "invert and multiply." For Example:

$$\frac{1}{6} \div \frac{2}{5} = \frac{1}{6} \times \frac{5}{2} = \frac{1 \times 5}{6 \times 2} = \frac{5}{12}$$

As students grow in both computational skills and mathematical maturity, they will omit using or recording several of the steps in the above algorithms.

The memory or central processing units of calculators and microcomputers are not programmed to perform operations on fractions in the way we commonly develop algorithms with middle grade students.

This problem can be circumvented on the microcomputer by programming it to find the answer using integer values of the fractions' numerators and denominators. Figure 1 presents a program and its "run" for finding the sum of two fractions and expressing the answer in lowest terms. The computer program is based on an integer definition for finding the sum of two fractions and Euclid's Algorithm for finding the greatest common fac-

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tor of two numbers. Through an adaptation of this computer program, we can devise a method which utilizes calculators in work with addition of fractions. This process will be explained shortly. First, a brief presentation of some old algorithms, no longer commonly taught, and an explanation of Euclid's Algorithm.

The following algorithms are sometimes referred to as definitions for operations on fractions. For clarification purposes, an algebraic expression of the rule is given and then applied in an appropriate computation.

Addition.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \qquad \frac{3}{4} + \frac{2}{5} = \frac{(3 \times 5) + (4 \times 2)}{(4 \times 5)} = \frac{15 + 8}{20} = \frac{23}{20}$$

Subtraction.

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \qquad \frac{3}{4} - \frac{2}{5} = \frac{(3 \times 5) - (2 \times 4)}{(4 \times 5)} = \frac{15 - 8}{20} = \frac{7}{20}$$

Multiplication.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \qquad \frac{3}{4} \times \frac{3}{5} = \frac{3 \times 3}{4 \times 5} = \frac{9}{20}$$

Division.

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \qquad \frac{2}{3} \div \frac{3}{5} = \frac{2 \times 5}{3 \times 3} = \frac{10}{9}$$

The above algorithms are used because they do not require finding the lowest common denominator in order to perform computations and, therefore,

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enable us to more easily apply the calculator in work with fractions. A careful reading of the above algorithms should lead one to conclude that the definitions of multiplication and division provide the patterns for the algorithms developed in today's textbooks for those two operations. Since it is sometimes desirable to express an answer in lowest terms, Euclid's Algorithm is introduced as a method for finding the greatest common factor of the numerator and denominator.

In essence, Euclid's Algorithm works in the following manner:

1. The larger of the two numbers is divided by the smaller.
2. If the quotient of the division is not an integer (i.e., the remainder is not zero), then the previous divisor becomes the new dividend of a new division and the remainder becomes the new divisor.
3. Steps one and two are repeated until a remainder of zero is obtained.
4. Once the remainder of zero is obtained, the GCF is determined--it's the last divisor.

Confused? Hopefully, the following application of Euclid's Algorithm for finding the GCF of 46 and 115 will clarify the procedure.

Divide the larger number by the smaller.

$$46\overline{)115}\quad 2,\ 92,\ 23$$

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The remainder (23) becomes the new divisor, and the previous divisor (46) becomes the new dividend.

$$23\overline{)46}\quad 2,\ 46$$

Twenty-three is the GCF. When the remainder is zero, that divisor is the GCF.

We now have the background necessary to effectively apply a calculator in work with fractions. Basically, there are four steps in utilizing a calculator in work with fractions:

1. Find the answer for the operation as a ratio of two integers.
2. Find the GCF of the numerator and denominator.
3. Divide the numerator and denominator by the GCF.
4. Write the answer in fractional form.

The following outline is used to illustrate an application of a calculator in finding the sum of two fractions. For each step, parenthesis are used to indicate the operation performed by the calculator.

STEP ACTION	RECORD ON PAPER
1. Write the problem using the definition of addition.	$\frac{3}{12} + \frac{5}{16} = \frac{3 \times 16 + 12 \times 5}{12 \times 16}$
2. Use calculator to find product of numbers in parenthesis. Record over denominator.	$\frac{3}{12} + \frac{5}{16} = \frac{(3\times16) + (12\times5)}{(12\times16)} = \frac{48+60}{192}$

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3. Use calculator to find sum of numerators. Record.

$$\frac{3}{12} + \frac{5}{16} = \frac{3x16 + 12x5}{12x16} = \frac{(48+60)}{192} = \frac{108}{192}$$

4. Use Euclid's algorithm to find the GCF of the numerator and denominator. Record only integer values in the quotients.

```
    1    (192÷108)          1    (108÷84)
108|192                    84|108
  108    (1x108)              84    (1x84)
   84    (192-108)            24    (108-84)

    3    (84÷24)            2    (24÷12)
24|84                      12|24
  72     (3x24)              24    (2x12)
  12     (84-72)              0    (24-24)
```

The fourth division has a remainder of zero, thus determining the GCF--12.

5. Divide numerator and denominator by the GCF. Record answer.

$$\frac{3}{12} + \frac{5}{16} = \frac{3x16 + 12x5}{12x16} = \frac{48+60}{192} = \frac{(108\div12)}{(192\div12)} = \frac{9}{16}$$

In a like manner, the calculator may be used to perform the number operations of multiplication, division and subtraction of fractions. It should be obvious that many of the above operations could have been per-

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formed without a calculator. As students develop skills for performing mental arithmetic and for using prime numbers and divisibility rules in work with fractions, they should need the calculator less. However, an understanding of procedures such as the above will provide mathematical enrichment and the beginning of a foundation for understanding computer applications in problem-solving. The primary purpose of this paper was to present a method for applying a calculator in work with fractions and to obtain an answer in fractional form.

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Figure 1

```
100 REM PROGRAM FOR ADDING FRACTIONS
110 REM
120 REM VARIABLES IN PROGRAM
130 REM   N1-NUMERATOR 1ST FRACTION
140 REM   N2-NUMERATOR 2ND FRACTION
150 REM   NA-NUMERATOR OF ANSWER
160 REM   D1-DENOMINATOR 1ST FRACTION
170 REM   D2-DENOMINATOR 2ND FRACTION
180 REM   DA-DENOMINATOR OF ANSWER
190 REM   LN-LARGEST NUMBER
200 REM   SN-SMALLEST NUMBER
210 REM    Q-INTEGER QUOTIENT OF DIVISION
220 REM    R-REMAINDER
230 REM  GCF-GREATEST COMMON FACTOR
235 REM
240 CLS:PRINT:PRINT
250 PRINT"THIS PROGRAM ADDS TWO FRACTIONS"
260 PRINT
270 PRINT"ENTER THE FOLLOWING DATA:"
280 PRINT"  NUMERATOR 1ST FRACTION";
290 INPUT N1
300 PRINT"  DENOMINATOR 1ST FRACTION";
310 INPUT D1
320 PRINT"  NUMERATOR 2ND FRACTION";
330 INPUT N2
340 PRINT"  DENOMINATOR 2ND FRACTION";
350 INPUT D2
360 REM
370 REM ROUTINE FOR SUM
380 REM
390 NA=N1*D2+D1*N2
400 DA=D1*D2
410 PRINT:PRINT
420 PRINT N1"/"D1"+"N2"/"D2"=";
430 REM
440 REM ROUTINE FOR GCF
450 REM
460 IF NA>=DA THEN LN=NA: SN=DA
470 IF DA>=NA THEN LN=DA: SN=NA
480 Q=INT(LN/SN)
490 R=LN-(Q*SN)
500 IF R=0 THEN GCF=SN:GOTO 580
510 LN=SN
520 SN=R
530 GOTO 480
540 REM
550 REM ROUTINE FOR REDUCING
560 REM AND PRINTING ANSWER
570 REM
580 NA=NA/GCF
590 DA=DA/GCF
600 PRINT NA"/"DA
610 END
```

```
RUN

THIS PROGRAM ADDS TWO FRACTIONS

ENTER THE FOLLOWING DATA
  NUMERATOR 1ST FRACTION ? 3
  DENOMINATOR 1ST FRACTION ? 12
  NUMERATOR 2ND FRACTION ? 5
  DENOMINATOR 2ND FRACTION ? 16

 3 / 12 + 5 / 16 = 9 / 16
```

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